99/102 Lumped mass model of Low and Langley – comparison of static and dynamic results for hanging catenary

1 Introduction

This article is a summary of a paper[1] presented at OMAE 2006. The authors have developed from first principles a lumped mass model incorporating both tension and bending. Their code can perform static, frequency domain and fully non-linear time domain calculations. The latter is carried out with the well-known Wilson-θ implicit integration scheme.

In order to benchmark the code the authors used a simple model of a flexible line, hanging in a catenary shape. The output of their code was compared with OrcaFlex and exact agreement was demonstrated.

2 Line data

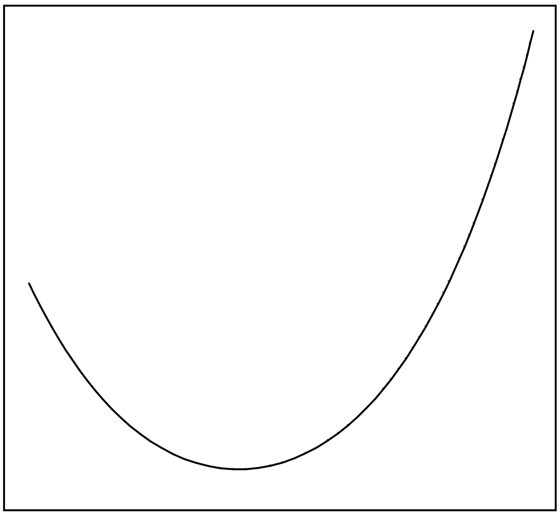

Figure1: Hanging catenary

The benchmark model used a line which was pinned at both ends. The horizontal and vertical distances between the top and bottom ends are 100m and 50m respectively and the top node is positioned at a water depth of 5m. An element length of 2.5m was used throughout the line.

Total unstretched length	170.0 m
Outer diameter	0.396 m
Dry mass	0.165 te/m
EA	500,000 kN
EI	120.8 kNm^2

[1] *Dynamic analysis of a flexible hanging riser in the time and frequency domain*, Ying Min Low and Robin Langley, University of Cambridge, UK, OMAE2006-92171.

Water density	1.0 te/m^3
Gravitational constant	9.807 m/s
Normal drag coefficient	1.0
Normal added mass coefficient	1.0

Table 1: Line data

3 Static results

The authors first compared results of a static analysis using their model against both OrcaFlex and the classical catenary equations. Agreement was exact with OrcaFlex. Very small differences are observed with the catenary equations since these do not account for elasticity. The authors observe that exact agreement with the catenary equations can be achieved by removing the bending springs from their model, a fact that can also be demonstrated in OrcaFlex by setting the bending stiffness to zero.

	Low & Langley	OrcaFlex	Catenary
Top tension (kN)	47.11	47.11	47.14
Bottom tension (kN)	26.60	26.60	26.63
Vertical reaction, top (kN)	45.71	45.71	45.72
Horizontal reaction, top (kN)	11.40	11.40	11.47
Vertical reaction, bottom (kN)	24.04	24.04	24.03
Horizontal reaction, bottom (kN)	-11.40	-11.40	-11.47

Table 2: Comparison of static results

The authors also performed a modal analysis using their model and compared the modal periods output by OrcaFlex, again achieving agreement.

4 Dynamic results

The authors compared the model to OrcaFlex for 6 different dynamic load cases. Cases 1 to 3 were in still water conditions with an oscilliatory harmonic motion imposed at the top end. Cases 4 to 6 had both ends fixed but imposed a linear, regular wave.

Case 1	Top motion	Surge, amplitude 10m, period 27s
Case 2	Top motion	Sway, amplitude 10m, period 27s
Case 3	Top motion	Heave, amplitude 10m, period 27s
Case 4	Single Airy Wave	Direction 0°, height 10m, period 10s
Case 5	Single Airy Wave	Direction 45°, height 10m, period 10s
Case 6	Single Airy Wave	Direction 90°, height 10m, period 10s

Table 3: Dynamic load case data

The authors present time history results of Effective Tension at the top end for each load case. Again, the agreement with OrcaFlex is exact. The time histories produced by OrcaFlex are included below for completeness. The authors show results from both their own code and OrcaFlex and all curves are completely coincident.

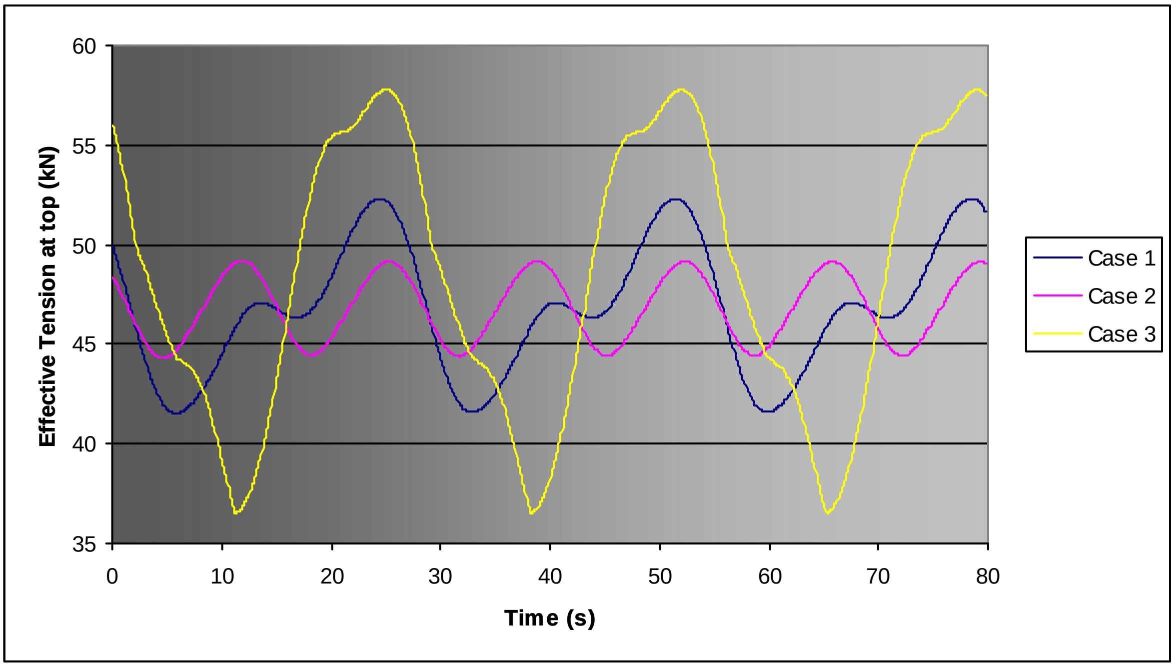

Figure 2: Effective Tension at top for cases 1, 2 and 3

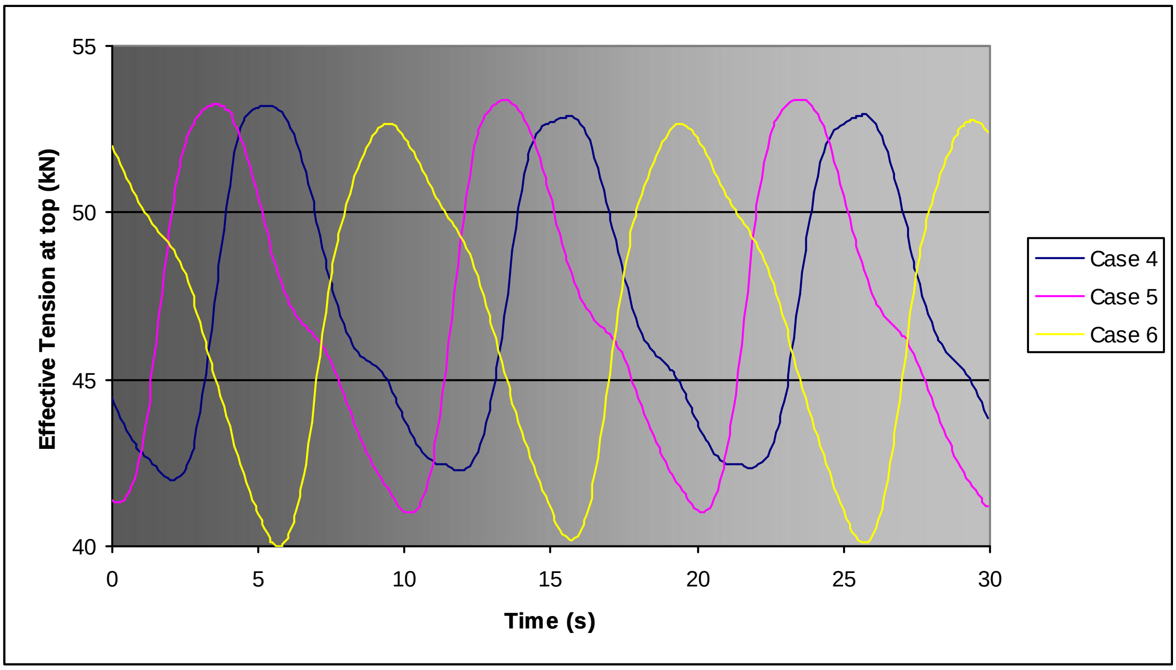

Figure 3: Effective Tension at top for cases 4, 5 and 6

5 Discussion

The authors have developed and implemented a lumped mass finite element model from first principles. This has been benchmarked against OrcaFlex for both static and dynamic analyses and complete agreement has been demonstrated.

The time integration scheme used by the authors is the Wilson-θ implicit scheme. The authors' OrcaFlex analyses used the OrcaFlex explicit scheme. OrcaFlex 9.0 introduced an implicit integration scheme (using the Newmark-β scheme) which gives the same results as the explicit scheme.