Strain reporting in OrcaFlex

1 Introduction

Consider a pipe with:

- Outside and inside radii r_o and r_i, and outside, inside and wall cross-sectional areas $A_o = \pi r_o^2$, $A_i = \pi r_i^2$ and $A_w = A_o - A_i$.
- Young's modulus E, axial stiffness $K_a = A_w E$ and Poisson ratio ν.
- External and internal pressures P_o and P_i.
- Wall tension T_w and effective tension $T_e = T_w + T_p$. Here $T_p = P_o A_o - P_i A_i$ is the pressure effect contribution to effective tension.

There are 4 sources of strain in OrcaFlex:

- Expansion strain $\epsilon_{expansion}$, specified by the user via the Expansion Factor on the line data form. The expansion strain will only be non-zero if the user specifies a value other than 1 for the expansion factor, which specifies a change in unstretched length and can either be a constant value or be time-variable.
- Poisson effect axial strain $\epsilon_{poisson}$ due to radial and circumferential strains caused by internal and external pressure. This is given by $\epsilon_{poisson} = 2\nu T_p/K_a = (2\nu T_p/A_w)/E$.
- Tensile strain $\epsilon_{tensile}$ caused by axial stress in the pipe wall (wall tension). It is given by $\epsilon_{tensile} = T_w/K_a = (T_w/A_w)/E$. Note that internal and external pressure influence the wall tension through the relationship between effective tension and wall tension (see above) and so contribute to the tensile stress and strain.
- Bending Strain $\epsilon_{bending}$. This is assumed to vary linearly across the cross-section, so it doesn't give any net length change.

2 Strain Results Reported by OrcaFlex

OrcaFlex reports the following results that give strain values.

Direct Tensile Strain	$=$	$\epsilon_{tensile}$ = tensile strain, due to wall tension only. This matches Direct Tensile Stress (i.e. Direct Tensile Stress = $E\times$ Direct Tensile Strain) and excludes any Poisson strain.
Max Bending Strain	$=$	$\epsilon_{bending}$ at the outer fibre
Worst ZZ Strain	$=$	Direct Tensile Strain $\pm$ Max Bending Strain where the + or - is whichever gives the large absolute value.
ZZ Strain	$=$	Direct Tensile Strain $+$ bending strain at specified (r, θ) position
	$=$	$\epsilon_{tensile} + \epsilon_{bending}$ at specified r, θ This matches ZZ Stress and excludes any Poisson strain.
Expansion Factor	$=$	$1 + \epsilon_{expansion}$

Note that the OrcaFlex strain results do *not* include any Poisson strain due to radial and circumferential stresses in the pipe wall caused by internal and external pressure, and do not include any expansion strain specified by an expansion factor. The OrcaFlex strain results therefore report strain relative to the expanded *pressured* unstretched length.

Note that length change contributions due to Poisson and expansion factor are not caused by axial stress. Poisson strain, for example, is caused by radial and circumferential stress due to internal and external pressure. The Poisson strain can be calculated from the OrcaFlex pressure results (Internal Pressure P_i and External Pressure P_o) using the formula

$$\epsilon_{poisson} = \frac{2\nu(P_o A_o - P_i A_i)}{K_a}$$

And the total length strain relative to the unpressured unstressed state can be obtained by adding this and any expansion strain $\epsilon_{expansion}$ to the Direct Tensile Strain.

3 Poisson Strain when using Non-linear Axial Stiffness

The above only considers the normal case of constant axial stiffness. This section explains how OrcaFlex handles the Poisson effect of pressure, with the explanation expressed in a way that also covers what happens if the user specifies the axial stiffness using a non-linear variable data item.

The axial stiffness data, whether linear or non-linear, effectively specifies the wall tension as a function of strain. OrcaFlex takes this data as specifying the wall tension as a function of strain when the line is in the *unpressured* state, i.e. at atmospheric external and internal pressure. When external or internal pressure is applied, the radial and circumferential stresses due to pressure give an axial strain due to the Poisson ratio effect, and Lamé's equation shows that the corresponding wall tension change is $-2\nu T_p$. So for a given stretched length the Poisson ratio effect of applying pressure is to subtract $2\nu T_p$ from the wall tension that would apply if the line was the same stretched length but unpressured.

This can alternatively be thought of as moving the origin of the curve relating wall tension to strain to the point where the tension is $2\nu T_p$, and then measuring both tensile strain and wall tension from zero at that point. This new origin is the point representing the new axially-unstressed length (i.e. zero axial stress) when *pressured*.